are travelling in the same direction. This can be seen in the following way: The propagation velocity of a forward-facing disturbance is $u_1 + c_1$; if $u_1 + c_1 > D$, the disturbance will overtake the shock. Equation (11) can be converted into a differential equation for the Hugoniot by assuming that $S = S(p, V)$ and proceeding to eliminate dS/dp. The result is:

$$\text{(15)} \qquad dp_1/dV = \{(\partial p_1/\partial V_1)_S + \Gamma(p_1 - p_0)/2V_1\}/\{1 - \Gamma_1(V_0 - V_1)/2V_1\},$$

where $\Gamma = V(\partial p/\partial T)_V/C_V$. It has already been shown that $-dp/dV > > (p - p_0)/(V_0 - V)$ in Fig. 2, and this with eq. (15) leads to the inequality:

$$\text{(16)} \qquad c_1^2/(D - u_1)^2 - \Gamma_1(V_0 - V_1)/2V_1 > 1 - (\Gamma_1/2V_1)(V_0 - V_1).$$

Then $c_1^2 > (D - u_1)^2$ or

$$\text{(17)} \qquad u_1 + c_1 > D_1,$$

provided the Rayleigh line is less steep than the tangent to the Hugoniot at (p_1, V_1). We say that in this case the flow behind the shock is ***subsonic***. A single shock connecting (p_0, V_0) and (p_1, V_1) is accordingly stable. If $D_1 > u_1 + c_1$, then (p_1, V_1) is a point of instability and the possibility of forming a second shock exists [2].

Experimentally produced shock waves seldom exactly satisfy the requirements of steady flow assumed in deriving the jump conditions. The states connected by the shock transition may not be precisely uniform or the shock wave has not propagated far enough to become steady. However, the experimental conditions may be very close to the theoretical assumptions, and it is quite likely that errors involved in applying the jump conditions to experiments are less than those originating from other sources. Such errors may be significant if gradients in adjacent regions are comparable to those in the shock or if time has not been sufficient for the flow to become steady and the curvature of the Hugoniot is large. The resolution of this question is a constant source of concern to experimentalists and no satisfactory resolution has been made. BLAND [3] has considered the development of a step change in pressure for a viscous material and concludes that the shock profile is essentially steady after travelling a distance of five shock thicknesses from the source. This is an interesting result. The difficulty in applying it is that, in general, the steady shock thickness is unknown.

2. – Rarefactions and characteristics.

Referring to our original model of a pressure on a half-space, we suppose that after being held at constant value p_1 while the shock was being formed, we then reduce the pressure to its ambient value p_0. A forward-facing rarefaction is produced, and we seek an appropriate method for describing the propagation of this rarefaction.

According to the discussion of Sect. 1, waves of rarefaction cannot be steady in the sense of eqs. (4)-(6) for normal materials; *i.e.*, there are no solutions of the form $\varrho = \varrho(x - Dt)$, etc., for constant D. We do know, however, that waves of infinitesimal amplitude satisfy the simple wave equation, and that solutions of this are in the form

$$f(x - ct) + g(x + ct)\,.$$

That is, they consist of forward-facing and backward-facing waves. For such infinitesimal waves we know that

$$\mathrm{d}p = \varrho c\,\mathrm{d}u \tag{18a}$$

for forward-facing waves and

$$\mathrm{d}p = -\,\varrho c\,\mathrm{d}u \tag{18b}$$

for backward-facing waves. Here c is the velocity with which these infinitesimal disturbances travel into material at rest. Equations (18) are in accord with the jump conditions if we apply them to waves of infinitesimal amplitude.

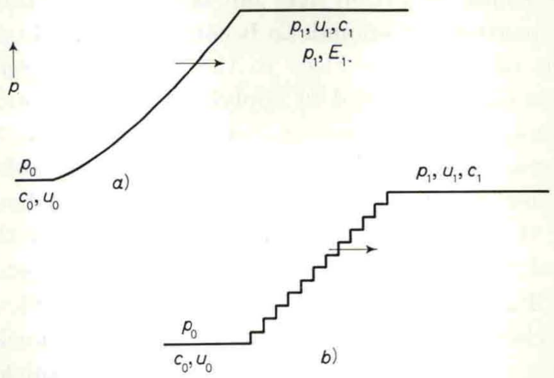

Fig. 3. – *a*) Continuous forward-facing rarefaction wave. The propagation velocity at each point is $u + c$. *b*) Representation of continuous rarefaction as sequence of small increments for which $\delta p = \varrho c\,\delta u$.

If we consider our forward-facing rarefaction to be represented by a sequence of jumps, $\mathrm{d}p$, to lower pressure, accompanied by a sequence of jumps, $\mathrm{d}u$, to smaller particle velocity, we can integrate eq. (18) to obtain the relation between p and u at any point in the rarefaction. This is illustrated in Fig. 3